JOURNAL OF MATERIALS SCIENCE 24 (1989) 1233–1242

Correlating the plastic strain ratio with ultrasonic velocities in textured metals

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A method for predicting the anisotropy of the plastic strain ratio from measurements of ultrasonic velocities is proposed. The calculations are based on the assumption that both the plastic strain ratio and the ultrasonic velocities display the statistical symmetries of the crystal orientation distribution function (ODF). The proposed formula is based on a theoretical calculation of the anisotropy of the R value and the ultrasonic velocities for 28 different textures.

1. Introduction

The ability to form metals into useful shapes is often dictated by their microstructure and texture. For planar anisotropic sheets, the plastic strain ratio, R, varies with the tensile direction and is associated with the earing behaviour in deep drawing applications.

The prediction of R anisotropy in textured metals has been studied extensively, resulting in considerable literature on this subject. The method most generally employed consists of calculating the Taylor factor $M(q)$ by means of the classical Bishop and Hill approach [1]. The orientation distribution function (ODF) is then used as a weighting factor in the prediction of the plastic property of interest. Such theoretical predictions lead to reasonably good agreement with the experimental plastic strain ratio, for example Bunge [2], Szpunar [3], and Semiatin *et al.* [4]; although Dabrowski *et al.* [5] observed fairly large discrepancies in an aluminium-killed steel.

The anisotropy of the plastic strain ratio can be obtained from texture data, but, because X-ray texture measurements are often difficult, attempts have been made to correlate the bulk ultrasonic velocities with the anisotropy of the R factor. Bussiere *et al.* [6] have assumed that there exists a linear relationship between R and the bulk, longitudinal and shear wave velocities. The resulting regression equations also contain terms involving the elastic constants.

A different approach towards a correlation of the plastic strain ratio and the anisotropy of ultrasonic wave velocities is proposed in this paper. We begin first by describing the theories used for the calculation of the R factor and ultrasonic velocity anisotropy, forming the basis of our subsequent studies.

2. Calculation of R-value anisotropy in textured metals

Quantitative theories of plasticity are being developed on the basis of the ODF formalism. Theoretical prediction of plastic deformation also requires a detailed knowledge of the operative mechanism of deformation as well as a consistent deformation theory. Such theories are generally classified as upper bound, like that of Taylor [7], or lower-bound, derived from a Sachs [8] approach. Theoretical predictions have already been tested on various materials and although the agreement with experiment is often satisfactory, deviations of more than 25% are not uncommon. One particular disadvantage of these theories is the lengthy and time-consuming calculations required.

Montheillet *et al.* [9] have recently proposed a theory known as the continuum mechanics of textured polycrystals (CMTP), which provides a simple way to quantify the texture and the plastic anisotropy relationship. This theory and its applications were examined and discussed by Lequeu [10] in his recent work and the reader is referred to the original paper [11] for the description of the method. It is assumed that the yielding behaviour of textured polycrystals can be expressed by a continuous yield function of the Hill [12] type. The anisotropy described by this function is referred to the $\langle 1\,0\,0 \rangle$ axes of the crystal representing the ideal orientation. In order to calculate the anisotropy of the plastic strain ratio a best fit determination of the yield function and the crystallographic yield surfaces has to be performed for bcc and fcc crystals.

The crystallographic yield surfaces contain information about the active slip system, usually $\{1\,1\,1\}$ $\langle 1\,1\,0 \rangle$ for fcc crystals and $\{1\,1\,0\}$ $\langle 1\,1\,1 \rangle$ for bcc metals. The yield function is then expressed in the specimen reference frame and then the plastic properties of interest can be expressed as a function of various ideal orientations. Such a mathematical treatment leads finally to the determination of the plastic strain ratio. There is a reasonable agreement between experiment and theory [10]. Theoretical calculations are, however, limited to materials which exhibit strong texture, justifying the assumption of an ideal orientation. More complex texture can be treated as a suitably weighted superposition of various ideal orientation components.

Following the method just described, the anisotropy of the R value in the plane of the sheet was calculated for 28 different ideal orientations in steel,

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aluminium, brass and copper. Textures were defined using the $(h\,k\,l)$ $[uvw]$ indices displaying a gaussian spread of 5°.

3. Ultrasonic velocities in textured materials

Sound is transmitted through solids in several ways, depending on the shape, size and structure of the material. In anisotropic materials, the velocity of sound varies with direction. Acoustic waves can propagate through the bulk or along the surface of solids. In this report only bulk waves are being considered. The calculation of the ultrasonic velocities involves the averaging of the elastic constants over the crystal orientation distribution function. Several possible methods can be employed for such calculations, the most common of which are due to Voigt, Reuss and Hill. Following the assumption that the stresses in every grain are the same, we obtain the Reuss approximation. The Voigt method assumes the uniformity of strain across the grain boundaries.

The first two methods give upper and lower bounds for the elastic constants. Kroner [13] and Morris [14] proposed a more realistic calculation but the results obtained are very close to the Hill average [15] . The method used for the velocity calculation follows the procedure described by Sayers [16] making use of polycrystal elastic constants.

Our calculations of the elastic constants were performed using the Hill approach and then the ultrasonic velocities were obtained as solutions of the Christoffel equation (Musgrave [17]). The same four metals and 28 different ideal orientations were analysed.

4. Method for predicting the anisotropy of the plastic-strain ratio from measurements of ultrasonic velocities

Both the anisotropy of the plastic strain ratio and of the ultrasonic velocities can be theoretically calculated for materials of known texture. The results of such predictions, however satisfactory, have still to be improved.

For the purpose of our discussion of the correlation between the plastic strain ratio and the ultrasonic velocities, we assume that the anisotropy of each is known. Such knowledge can be based on experiment or, as here, be derived from theory. The main argument for the use of theoretical values for the anisotropy is the difficulty in obtaining materials having a variety of theoretically possible textures. Such a variety is necessary for verifying a model involving correlations of the anisotropy of various properties.

Theoretically, both crystal symmetry and orientation distribution function (ODF) symmetries are responsible for the anisotropy of physical properties. It is well known (Neumann's Principle) that the symmetry elements of any physical property must include the symmetry elements of the point group of the crystal. The crystal symmetry may, therefore, decide the anisotropy of physical properties. In a similar manner, texture-related properties should also display the statistical symmetries of the ODF.

Mathematically, the distribution of crystallites in a textured material is described by the following function.

$$f(\psi, \phi, \psi_2) = \sum_{l=0}^{\infty} \sum_{\mu=1}^{M(l)} \sum_{\nu=1}^{N(l)} C_l^{\mu\nu}\, \dot{T}_l^{\mu\nu}(\psi, \phi, \psi_2)$$

where $(\psi_1, \phi, \psi_2) \equiv g$ are the Euler angles defining the orientation of the grain in the specimen reference frame. The $C_l^{\mu\nu}$ are expansion coefficients and the $\dot{T}_l^{\mu\nu}$ are generalized spherical harmonics, symmetrized according to the specimen and crystal symmetries. Physically, the $C_l^{\mu\nu}$ represent the volume fraction of grains displaying the symmetry defined by the $\dot{T}_l^{\mu\nu}(g)$.

If texture is the only reason for anisotropy, the symmetry of the physical properties is related to the symmetries represented by the low order $T_l^{\mu\nu}(g)$ texture functions. For example, if the grain orientation distribution has a four-fold axis, so should the physical property. The strength of the four-fold anisotropy will, therefore, be represented by the value of the corresponding $C_l^{\mu\nu}$ coefficient. Of particular importance are the coefficients C_4^{12} and C_4^{14} which describe the texture contributions to second- and fourth-order terms in the anisotropy of the physical property. A linear relationship should, therefore, exist between the order of the anisotropy of the physical property and the $C_4^{1\nu}$ texture coefficient which describes this order.

Bunge [2] has demonstrated that such a relationship exists between the elastic anisotropy of a rolled copper sheet and the corresponding $C_4^{1\nu}$ coefficients. However, in the case of the R-value and ultrasonic velocity anisotropy the complexity of modelling plastic behaviour makes it extremely difficult to derive an analytical formula to demonstrate the existence of such a linear relationship. A different approach must therefore be suggested.

From experiment it is known that both the ultrasonic velocities, V, and the R-value are slowly varying functions in the plane of the specimen and therefore both can be well represented as Fourier series. Hence, to a good approximation,

$$V(\alpha) \cong V_0 + \sum_{\nu=1}^{\nu=N} (V_\nu \cos \nu\alpha + W_\nu \sin \nu\alpha)$$

$$R(\alpha) \cong R_0 + \sum_{\nu=1}^{\nu=N} (R_\nu \cos \nu\alpha + P_\nu \sin \nu\alpha)$$

where $V(\alpha)$ and $R(\alpha)$ are evaluated in the plane of the sheet at an angle α from the rolling direction. If there exists a linear relationship between the V_ν and $C_l^{1\nu}$ and also between the r_ν and $C_l^{1\nu}$, then there should be a linear relationship between the V_ν and r_ν.

For all applications where rolling symmetry exists, the sin (νd) terms must be omitted. Furthermore, the odd cosine terms must also be omitted as a result of symmetry with respect to the transverse direction.

Symmetry will therefore simplify the correlations between plastic behaviour and the ultrasonic velocities to the consideration of the even cosine coefficients of the expansions

$$R(\alpha) = R_0 \left(1 + \sum_{\nu=1}^{M} r_{2\nu} \cos(2\nu\alpha)\right)$$

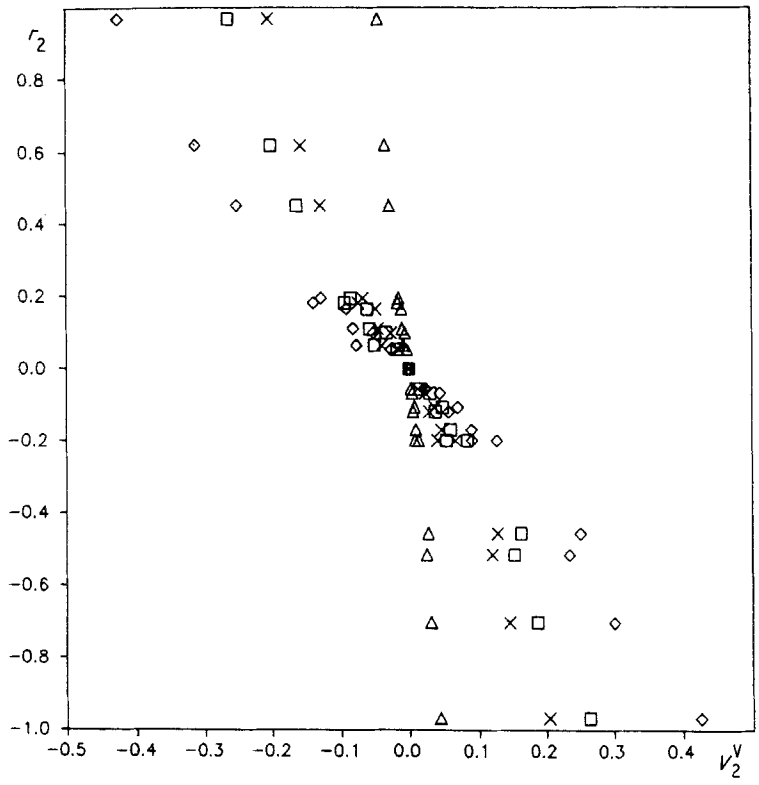


Figure 1 Relationship between the second-order Fourier coefficients of the plastic strain ratio (r_2) and the vertical shear velocity (V_2^V) for 28 textures of (□) copper, (△) aluminium, (◇) brass and (×) iron.

$$V(\alpha) = V_0\left(1 + \sum_{\nu=1}^{M} v_{2\nu} \cos(2\nu\alpha)\right)$$

Having presented a strong indication for a linear correlation between the same order coefficients of the plastic strain ratio and ultrasonic velocities, we proceed to study its realization for the chosen theoretical models.

In this work the theoretical values of the ultrasonic velocities and the plastic strain ratio were calculated in various directions on the specimen surface. These theoretical curves were represented as Fourier series and the series expansion coefficients of the same order were plotted for 28 different textures. The linear relationship was assumed between the corresponding Fourier components of the plastic strain ratio and the ultrasonic velocities. Each of the three velocity polarizations were considered: V^L, the polarization along the direction of propagation; V^H, the polarization direction perpendicular to the direction of propagation but in the plane of the specimen; and V^V, the polarization perpendicular to the propagation direction and perpendicular to the specimen surface. Table I gives the slopes m for the appropriate relationships with the correlation coefficients

$$r = \frac{\overline{xy} - \bar{x}\bar{y}}{[(\overline{x^2} - \bar{x}^2)(\overline{y^2} - \bar{y}^2)]^{1/2}}$$

used as a measure of how well the correlations hold ($r = \pm 1$ for a perfect fit and $r = 0$ if there is no correlation). Zero-, second-, fourth- and six-order Fourier series expansion coefficients were considered (see Table I).

TABLE I Slopes (m) and correlation coefficients (r) for the correlation of the Fourier coefficients of the strain ratio $R(\alpha)$ with the velocities $V^i_{(\alpha)}$ for iron and aluminium

Order of Fourier coeffs	Velocity polarization		
	Longitudinal	Horizontal	Vertical*
Iron			
2	$m = 13.8 \pm 0.7$ $r = 0.9646$	$m = 61.5 \pm 11.5$ $r = 0.7219$	$m = -4.19 \pm 0.13$ $r = -0.9874$
4	$m = -12.0 \pm 0.5$ $r = -0.9794$	$m = 3.55 \pm 0.15$ $r = 0.9775$	–
6	$m = 57.2 \pm 4.0$ $r = 0.9430$	$m = -22.6 \pm 1.4$ $r = -0.9501$	–
Aluminium			
2	$m = 87.7 \pm 3.2$ $r = 0.9834$	$m = 2130 \pm 380$ $r = 0.7402$	$m = -19.4 \pm 0.6$ $r = -0.9865$
4	$m = -74.5 \pm 2.8$ $r = -0.9821$	$m = 16.5 \pm 0.6$ $r = 0.9818$	–
6	$m = 2285 \pm 120$ $r = 0.9670$	$m = -702 \pm 33$ $r = -0.9731$	–

*–, no correlation.

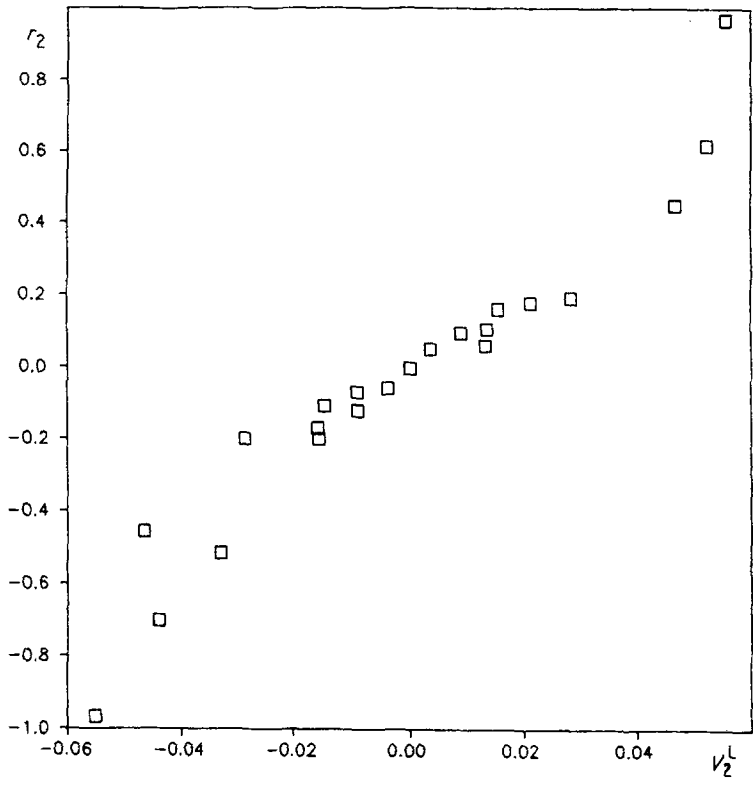


Figure 2 Graphical representation of the second-order Fourier coefficients of the plastic strain ratio (r_2) and the longitudinal velocity (V_2^L) for 28 textures of copper.

Excellent correlation was found between r_2 and V_2^L and r_2 and V_2^V. Also a very good correlation was found to exist between r_4 and V_4^L and V_4^H. Sixth-order Fourier series expansion coefficients of R and V^L and V^H also correlated well. The above listed conclusions were valid for all four metals investigated. Only the second-order Fourier series expansion coefficients of V^V correlated well with the coefficients of plastic-strain ratio. The rest did not, as one can see in Table I.

To summarize the results, there was good evidence that the correlation between the following components of the strain ratio R and the corresponding components of ultrasonic velocities exists:

$$r_2 = m_2^L V_2^L$$
$$r_2 = m_2^H V_2^H$$
$$r_2 = m_2^V V_2^V$$
$$r_4 = m_4^L V_4^L$$
$$r_4 = m_4^H V_4^H$$
$$r_6 = m_6^L V_6^L$$
$$r_6 = m_6^H V_6^H$$

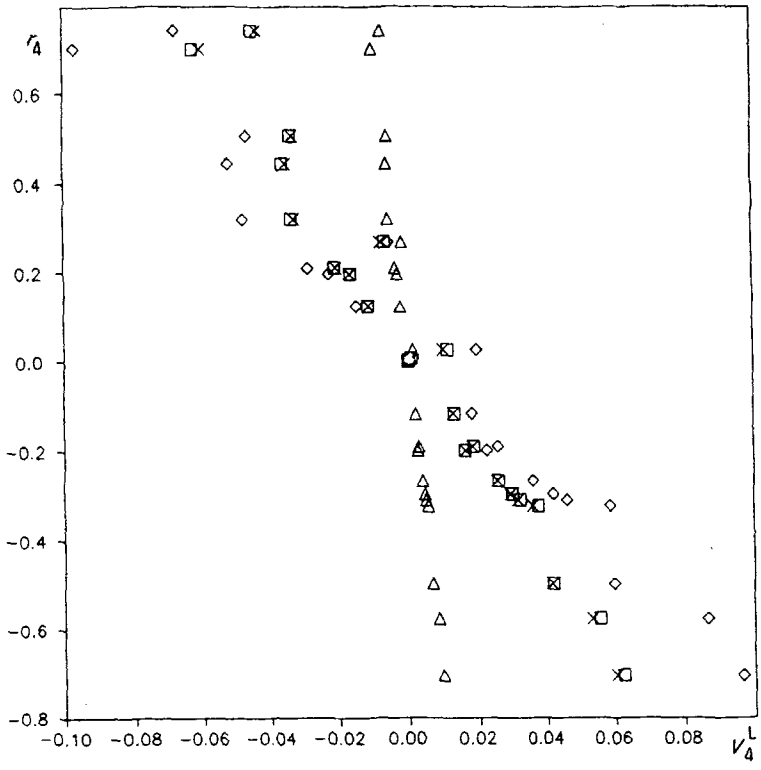


Figure 3 Relationship between the fourth-order Fourier coefficients of the plastic strain ratio (r_4) and the longitudinal velocity (V_4^L) for 28 textures of copper, aluminium, brass and iron.

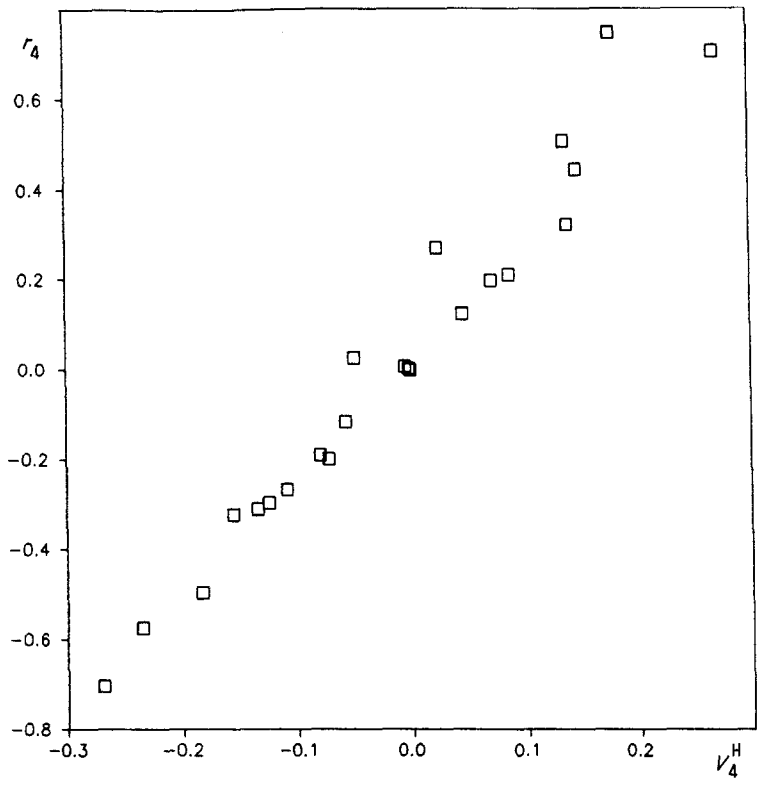


Figure 4 Details of the correlation of the fourth-order Fourier coefficients of the plastic strain ratio (r_4) and the horizontal shear velocity (V_4^H) for 28 textures of copper.

One can conclude from these results that there are three different ways to extract the second-order Fourier coefficient for the plastic strain ratio, and two independent ways to extract the fourth- and sixth-order series expansion coefficients from velocity data.

For the purpose of applying the correlation results to the prediction of the angular variation of R, the effect of the size of the series expansion coefficients and the accuracy of their determination is of considerable importance.

As already mentioned, the second-order coefficients of the ultrasonic velocities V^L and V^V correlate very well with the corresponding coefficients of the plastic strain ratio (for aluminium $r = 0.83$ and $r = 0.98$, respectively).

The correlation for V_2^V is illustrated in Fig. 1 for the four different metals. Additionally, the longitudinally polarized velocity coefficient V_2^L for copper is related to the corresponding r_2 coefficient as shown in Fig. 2. The second-order term of the velocity with horizontal polarization does not correlate well with plastic strain ratio, having a correlation coefficient of about 0.7.

There are also two possibilities to derive the fourth-order coefficients of the plasticity but, in contrast to

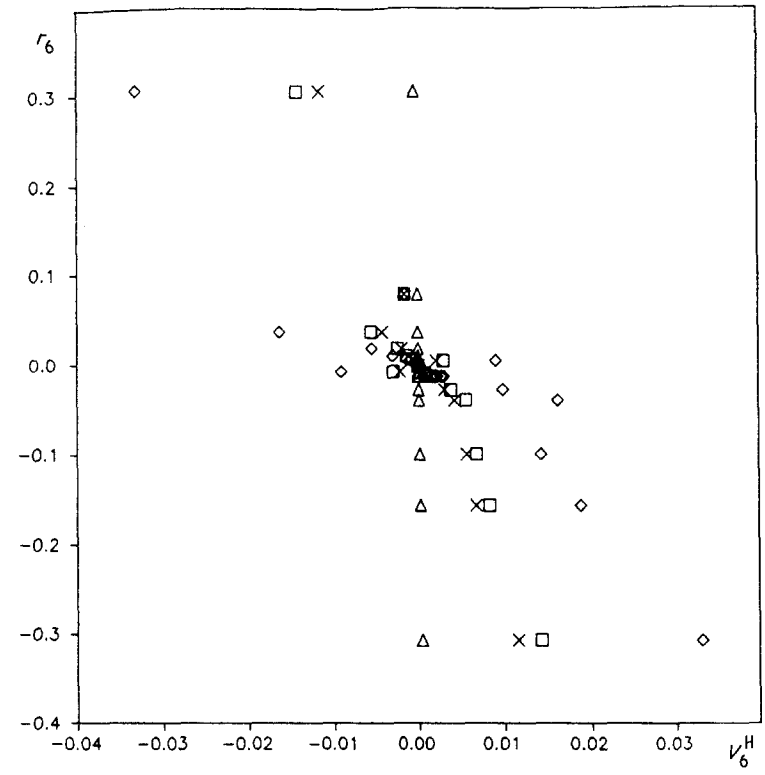


Figure 5 Correlation of the sixth-order Fourier coefficients of $R(\alpha)/R_0$; and $V^H(\alpha)/V_0^H$ for 28 textures of (□) copper, (△) aluminium, (◇) brass and (x) iron.

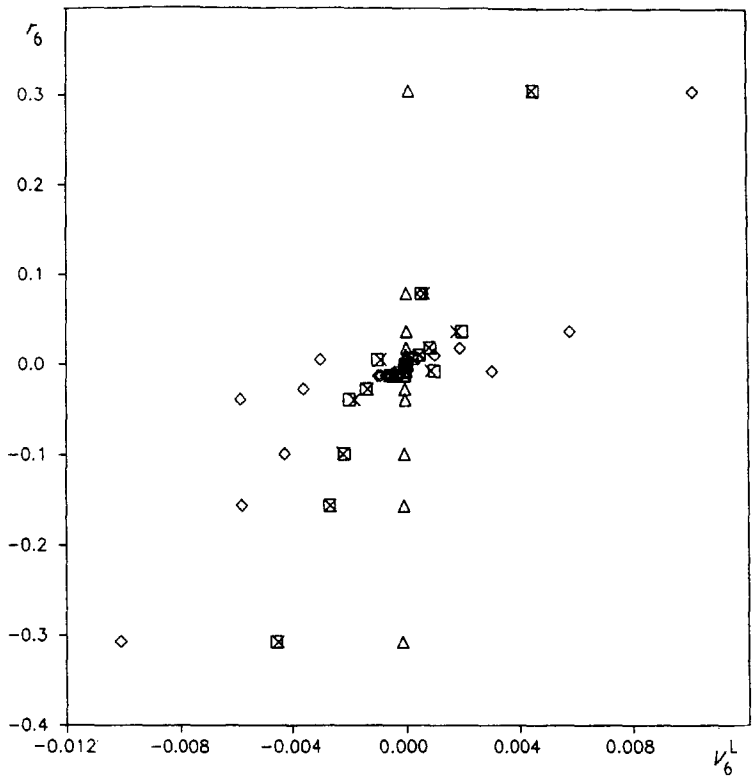


Figure 6 Graphical representation of the sixth-order Fourier coefficients of the plastic strain ratio (r_6) and the longitudinal velocity ($V_6{}^L$) for 28 textures of (□) copper, (△) aluminium, (◇) brass and (x) iron.

the second-order coefficient, the vertical shear velocity cannot be used. Instead, the series coefficients V_4^L and V_4^H might be applied. Fig. 3 illustrates the correlation between r_4 and V_4^L for the four metals and in Fig. 4 the relationship between r_4 and V_4^H is presented.

The sixth-order coefficients are, in general, much smaller than those for the fourth-order. The correlation of the plastic strain ratio with V_6^H and V_6^L is not as good as for the second- and fourth-order coefficients, r being between 0.88 and 0.97. V_6^V cannot be used at all for the purpose of correlation (see Figs 5 and 6).

From this discussion one can conclude that there exist several ways to predict the changes of the R-value using correlated Fourier series expansion coefficients. One way is to measure the ultrasound velocity having a polarization vector parallel to the propagation direction. The velocities having other polarizations cannot be used individually, because the accuracy of prediction using V^H or V^V only is too low. This is illustrated in Fig. 7a where the variation of R value with α was predicted using Fourier coefficients for each of the ultrasonic velocities independently, according to the formula

$$\frac{R(\alpha)}{R_0} = 1 + m_2 V_2 \cos 2\alpha + m_4 V_4 \cos 4\alpha + m_6 V_6 \cos 6\alpha$$

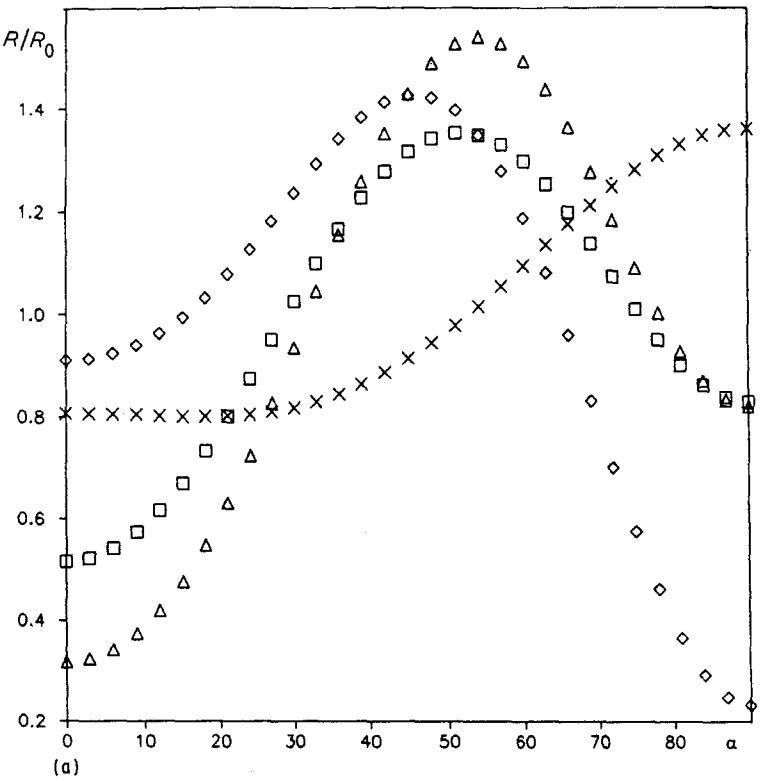
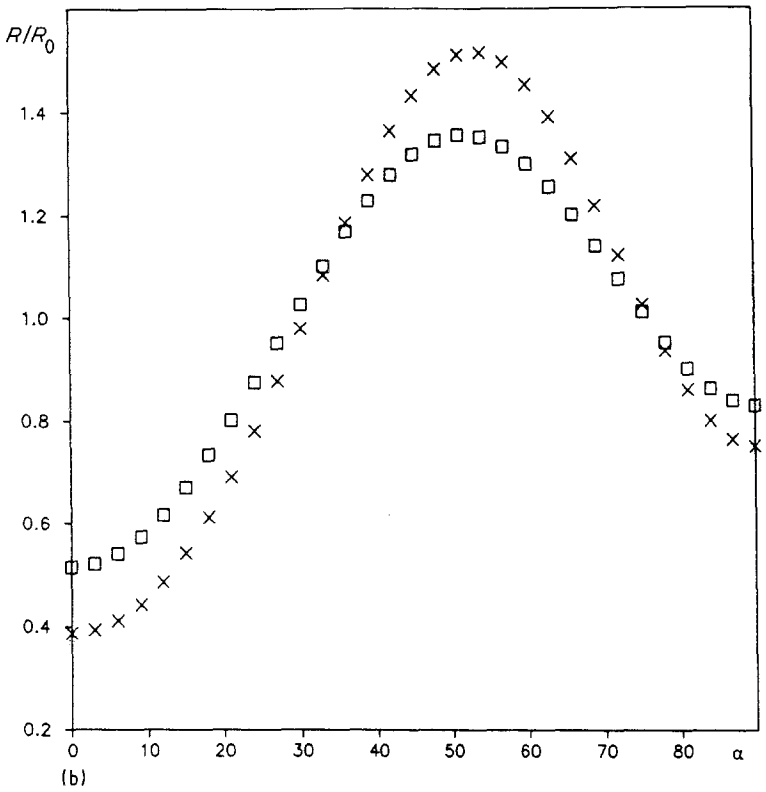


Figure 7 Predictions of the anisotropy of the plastic strain ratio $R(\alpha)/R_0$ for the texture (1 1 0) [1 $\bar{1}$ 2] based (a) on independent correlations with the velocities (△) V^L, (◇) V^H and (x) V^V, and (b) on an optimal mixed correlation (x). Results are compared to the source data values (□).

Figure 7 Continued



Additional typical results are compared to the theoretical changes of $R(\alpha)$ in Figs 8a, 9a and 10a. Clearly, the longitudinal velocity gives the best prediction.

The other velocity polarizations may give good estimations of the R-value anisotropy for certain types of texture; however, they occasionally may generate unrealistic results. Such cases are demonstrated in Fig. 10a where the prediction of $R(\alpha)$ for the texture (1 0 0) [0 0 1] and (1 2 3) [1 $\bar{2}$ 1], from V^{V} and V^{H} respectively, leads to erroneous results.

Only V^{L} ultrasonic waves gave a consistently good prediction of the R anisotropy in the plane of the sheet for all the examined textures.

There is, however, still a good possibility of obtaining further improvement in the accuracy of the prediction by using coefficients obtained from a mixture of various ultrasonic waves. As we have already mentioned, the V^{V} and V^{H} ultrasonic waves cannot be used individually to predict the anisotropy of R, but very good agreement can be obtained from the following mixture of V^{V} and V^{H} coefficients.

$$\frac{R(\alpha)}{R_0} = 1 + m_2^{V} V_2^{V} \cos 2\alpha + m_4^{H} V_4^{H} \cos 4\alpha + m_6^{H} V_6^{H} \cos 6\alpha$$

This expression allows us to predict with improved

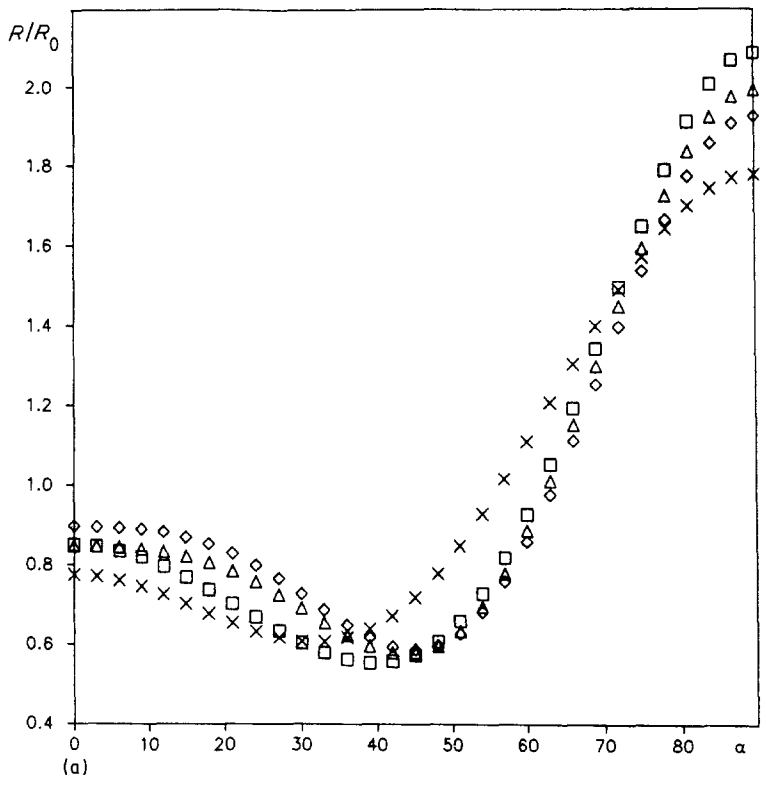


Figure 8 Predictions of the anisotropy of the plastic strain ratio $R(\alpha)/R_0$ for the texture (1 2 3) [5 $\bar{1}$ $\bar{1}$] based (a) on independent correlations with the velocities (△) V^{L}, (◇) V^{H} and (x) V^{V}, and (b) on an optimal mixed correlation (x). Results are compared to the source data values (□).

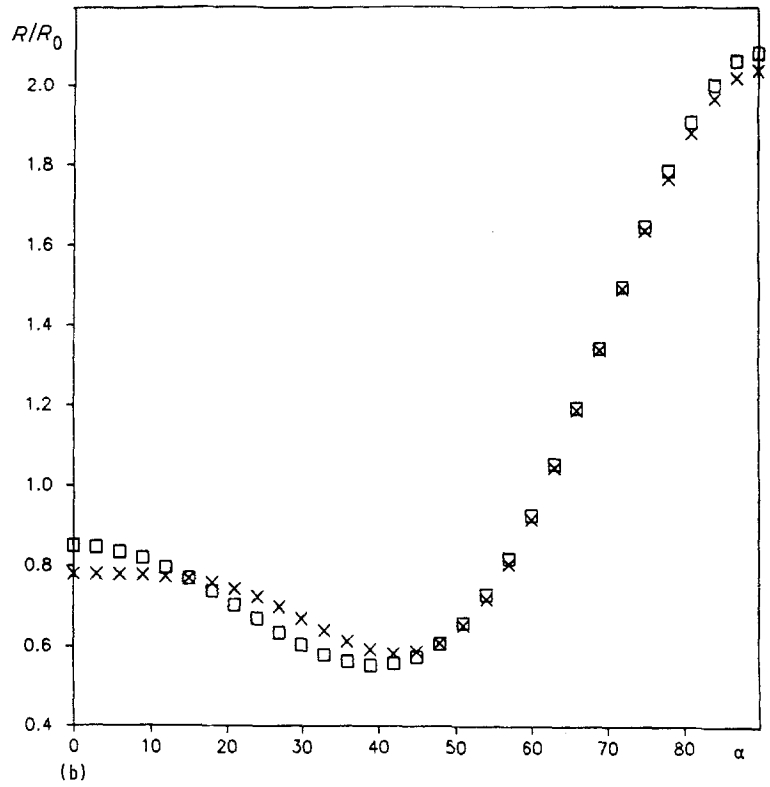


Figure 8 Continued

accuracy the anisotropy of R for all 28 chosen textures. Examples are provided in Figs 7b, 8b, 9b, and 10b. This selection of velocity coefficients was made taking into account the best fit between the corresponding r_v and V_v coefficients as well as the magnitude of their contribution to the relevant ultrasonic velocity. Explicitly, the equation is composed of terms involving V_2^V, V_4^V and V_6^H which do not exhibit better correlation than the corresponding V^L terms. In these cases, however, the appropriate slopes, m (see Table I), are much lower, implying that the size of ultrasonic velocity coefficients are higher than those of V^L, and therefore more easily determined.

All predictions for the anisotropy of the plastic strain ratio have been expressed in the relative units $R(\alpha)/R_0$, where R_0 is the zero-order coefficient of the Fourier series expansion, representing the average value. A question that might be asked is whether it is possible to predict this average value of R from ultrasonic measurements. Texture is responsible for anisotropy so that in principle only the anisotropy of R can be derived from the anisotropy of the ultrasonic

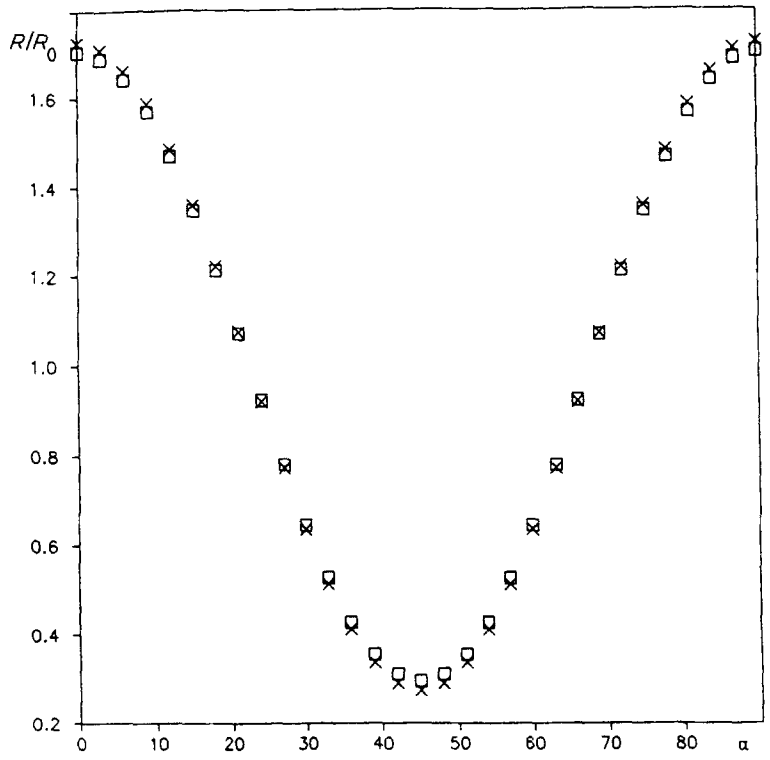
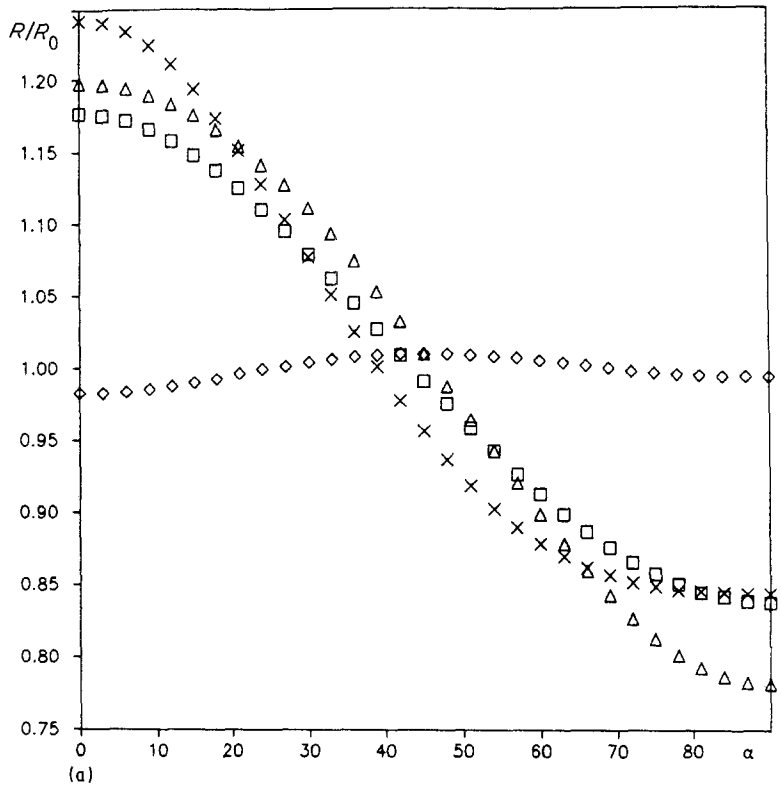


Figure 9 Predictions of the anisotropy of the plastic strain ratio $R(\alpha)/R_0$ for the texture (1 0 0) [0 0 1] based on an optimal mixed correlation (x). Results are compared to the source data values (□).



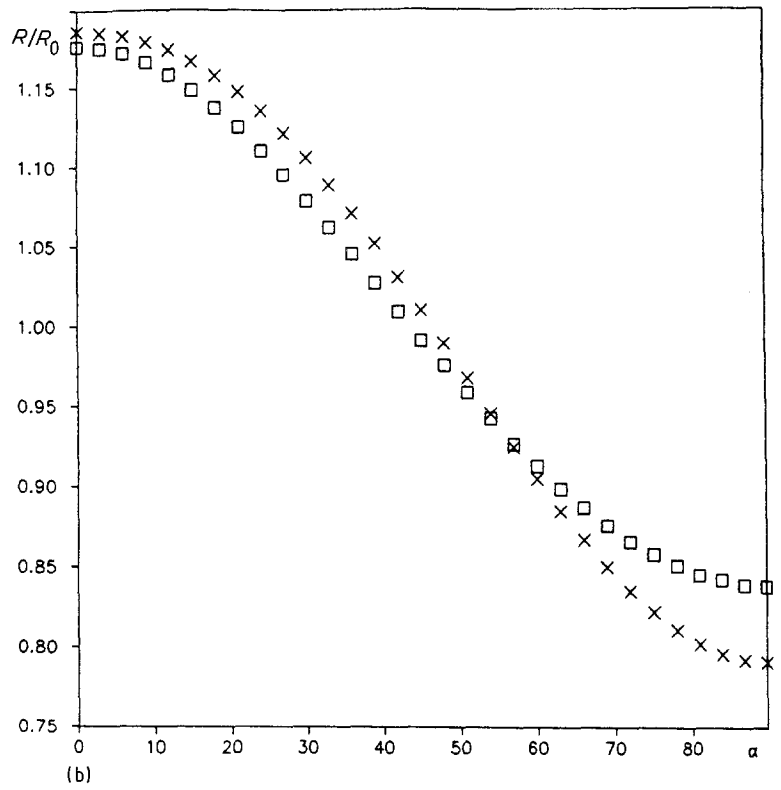


Figure 10 Predictions of the anisotropy of the plastic strain ratio $R(\alpha)/R_0$ for the texture (1 2 3) [1 $\bar{2}$ 1] based (a) on independent correlations with the velocities (△) V^L, (◇) V^H and (x) V^V, and (b) on an optimal mixed correlation (x). Results are compared to the source data values (□).

velocities, not the average value. However, R_0 for various selected textures also displays some relation to the average value of the ultrasonic velocity. This relationship is illustrated in Fig. 11 for iron and aluminium, where a linear relationship exists only for a limited interval of velocities.

For a vertical shear velocity lower than 3200 m sec^{-1} in steel and a longitudinal velocity higher than 6585 m sec^{-1} in aluminium the functional relationship between the average R-value and the average velocity fails to hold and a sort of branching phenomenon is observed. Attempts to correlate R_0 with V_0^H for the different metals were also not fully successful. Nevertheless, a limited correlation was observed which might be valid for the estimation of R_0 from a restricted interval of ultrasonic velocity measurements.

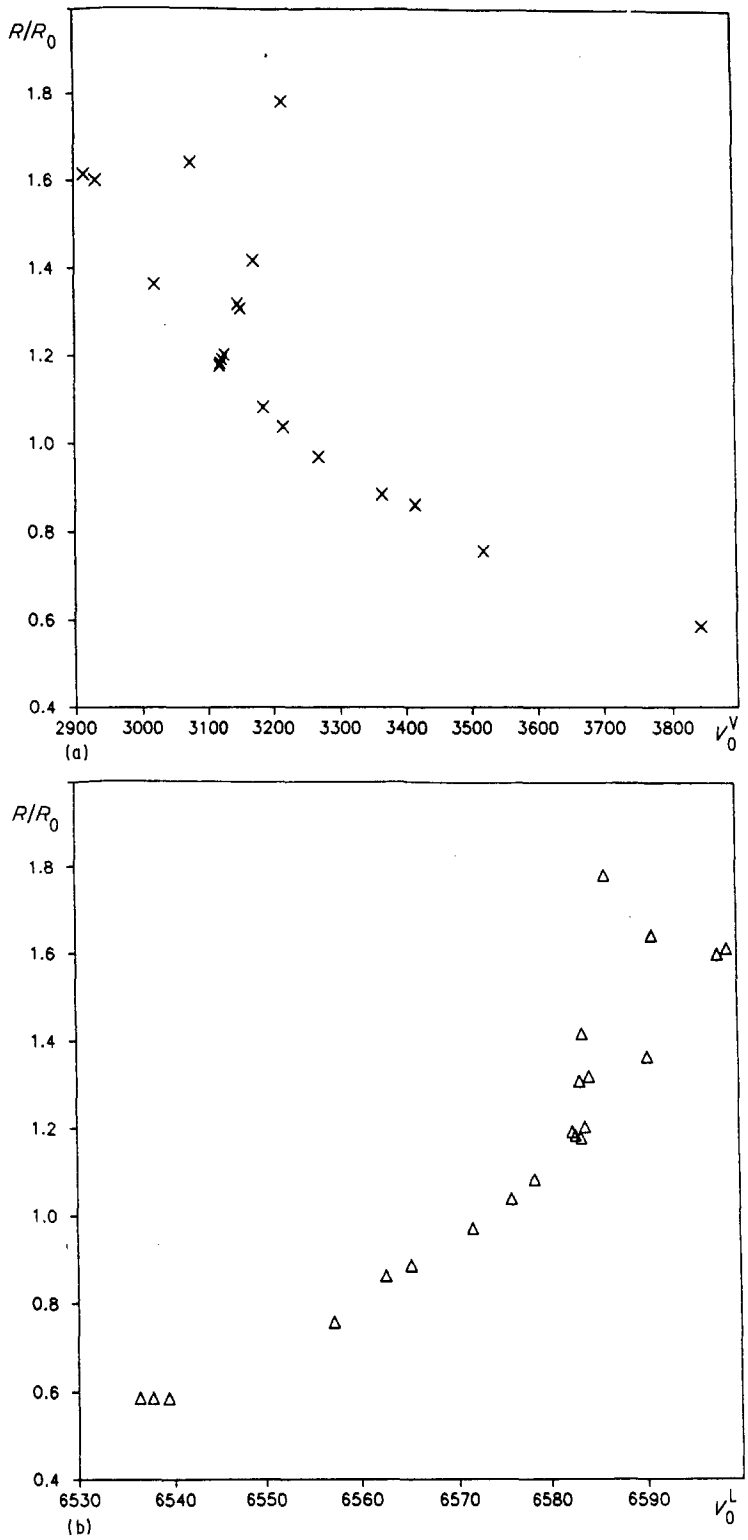


Figure 11 Graphical representation of the zero-order Fourier coefficients of the plastic strain ratio (R_0) with (a) the vertical shear velocity (V_0^V) in iron and with (b) the longitudinal velocity (V_0^L) in aluminium.

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Received 7 December 1987
and accepted 29 April 1988